TRIG. FORMULAE NOTES

MODULE 8: Sine- Cosine- and Area Rule

1. **Area rule** (S∠S)

$$\text{Area } \triangle ABC = \frac{1}{2} ab \sin C$$

2. **Sine rule** (SS∠), (∠∠S)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

3. **Cosine rule** (SSS), (S∠S)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

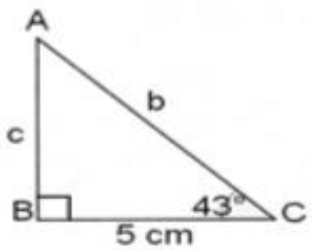
and

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Examples (Solving Δ's)

1. **In right-angled Δ:**
(use trig. ratios)

Determine b and c



1.1 $\frac{c}{5} = \tan 43°$

$\therefore c = 5 \tan 43° = 4{,}7 \text{ cm}$

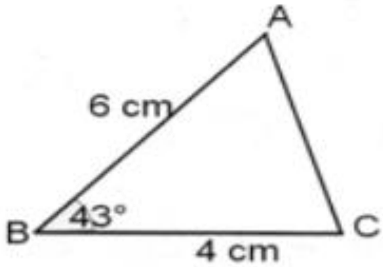
1.2 $\frac{5}{b} = \cos 43°$

$\therefore b = \frac{5}{\cos 43°} = 6{,}8 \text{ cm}$

Examples

In **non** right-angled **Δ's**

2. **Area rule:** (S∠S)

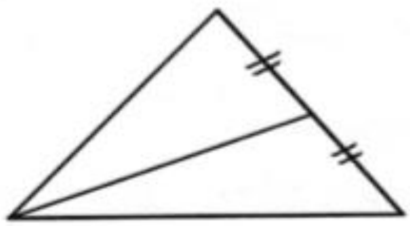


$$\text{Area } \triangle ABC = \frac{1}{2} ac \sin B$$

$$\text{area.} = \frac{1}{2} \times 4 \times 6 \sin 43°$$

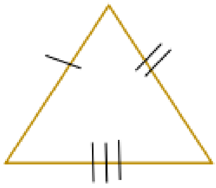
$$= 8{,}2 \text{ cm}^2$$

median – divide area of Δ in two equal parts

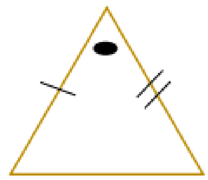


<u>USE COSINE WHEN YOU ARE GIVEN:</u>

1. SSS

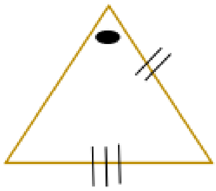


2. SAS (TWO SIDES AND AN INCLUDED ANGLE)

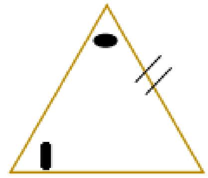


<u>USE SINE WHEN YOU ARE GIVEN:</u>

1. SSA



2. AAS



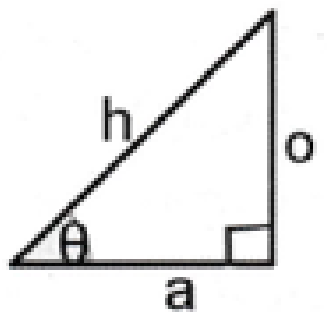
<u>RIGHT ANGLED TRIANGLE</u>

<u>USE TRIG FUNCTIONS:</u>

$$\sin\theta = \frac{o}{h}$$

$$\cos\theta = \frac{a}{h}$$

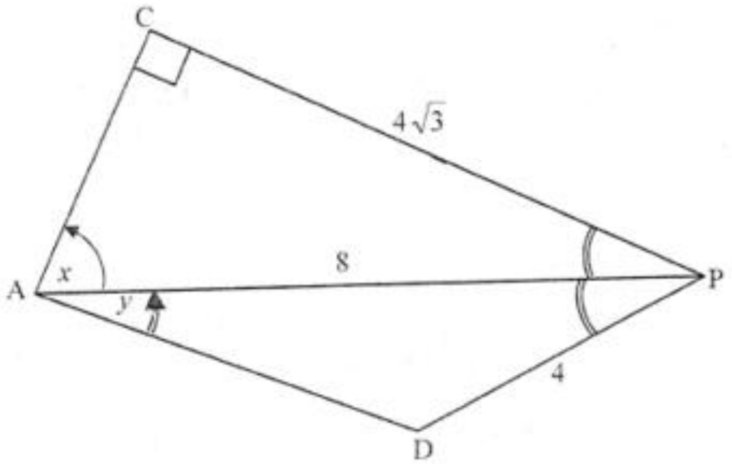
$$\tan\theta = \frac{o}{a}$$



TRIGONOMETRY NOTES

QUESTION 5

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.



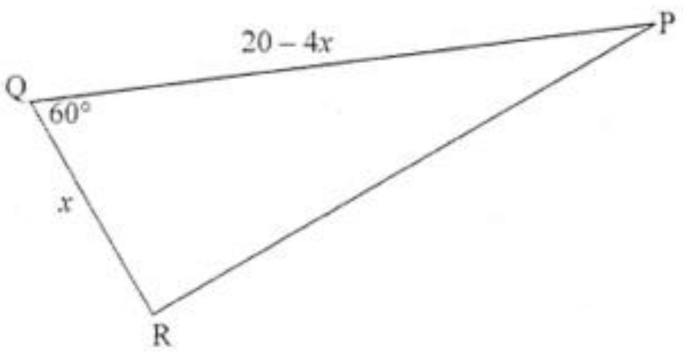
5.1	Show, by calculation, that $x = 60°$.	(2)
5.2	Calculate the length of AD.	(4)
5.3	Determine y.	(3)
		[9]

5.1	$\sin C\hat{A}P = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$ OR/OF $\frac{\sin 90°}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60°$

5.2	$C\hat{P}A = D\hat{P}A = 30°$ (AP bisects $D\hat{P}C$) $AD^2 = AP^2 + DP^2 - 2.AP.DP.\cos A\hat{P}D$ $= 8^2 + 4^2 - 2(8)(4)\cos 30°$ $= 8^2 + 4^2 - 2(8)(4)(\frac{\sqrt{3}}{2})$ $= 24,57...$ $AD = 4,96$
5.3	$\frac{\sin D\hat{A}P}{DP} = \frac{\sin A\hat{P}D}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30°}{4,96}$ $\sin y = \frac{4\sin 30°}{4,96}$ $= 0,403...$ $y = 23,78°$

QUESTION 7

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.



7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

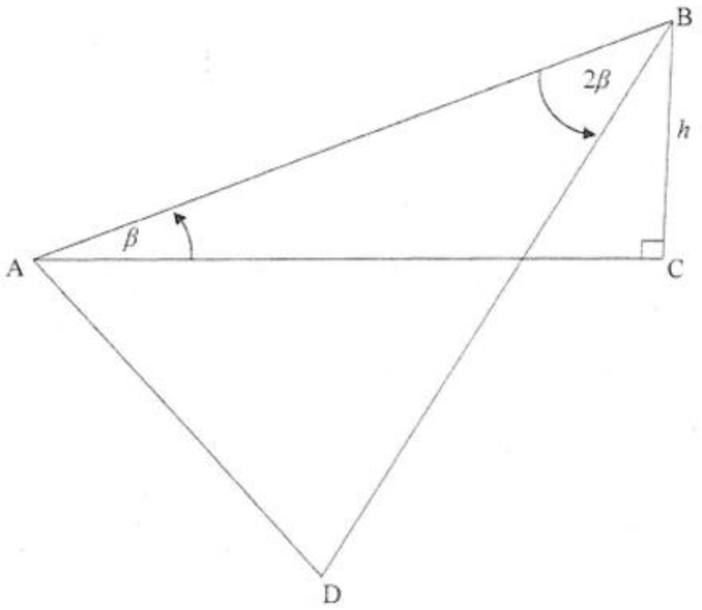
7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* $\Delta PQR = \frac{1}{2}PQ.QR.\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓subst into area rule/ *subst in opp-reël* ✓subst & simpl/ *subst en vereenv* (2)
7.1.2	For maximum area/*Vir maksimum opp:* $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore x_{max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* 2,5	✓$(\text{Area } \Delta PQR)' = 0$ ✓$5\sqrt{3} - 2\sqrt{3}x$ ✓answ/*antw* (3)
	OR/*OF* $x_{max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2}$ or $2\frac{1}{2}$ or 2,5	✓formula/*e* ✓ subst ✓answ/*antw* (3)
	OR/*OF* $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5-x) = 0$ $\therefore x = 0$ or 5 $\therefore x_{max} = \frac{0+5}{2} = \frac{5}{2}$ or/*of* 2,5	✓x-intercepts/ *x-afsnitte* ✓ subst ✓answ/*antw* (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2{,}5^2 - 2(10)(2{,}5)\cos 60°$ $= 81{,}25$ $\therefore RP = 9{,}01$	✓subst into cosine rule/*in cos-reël* ✓simpl/*vereenv* ✓answ/*antw* (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and BA = BD.



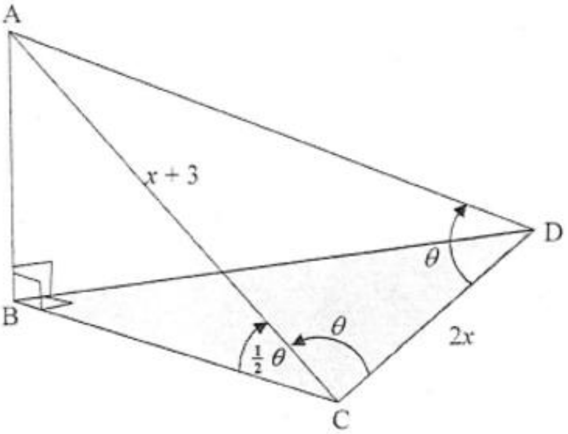
Determine the distance AD between the two anchors in terms of h. (7)

[15]

7.2	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: AB = BD and/*en* $A\hat{D}B = 90° - \beta$ [$\angle$s of/*v* $\Delta = 180°$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90° - \beta)}{AB}$ $AD = \frac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{h}{\sin\beta} \times \frac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$	✓AB ito h and/*en* β ✓ $A\hat{D}B = 90° - \beta$ ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓AD as subject/ *onderwerp* ✓expansion/*uitbrei* ✓$\sin(90° - \beta) = \cos\beta$ ✓answer ito h (7)
	OR/*OF* In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: AB = BD $AD^2 = AB^2 + AB^2 - 2AB.AB.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2(1 - 2\sin^2\beta)$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore AD = 2h$	✓AB ito h and/*en* β ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓expansion/*uitbrei* ✓multiplication/ *vermenigv* ✓simpl/*vereenv* ✓answer ito h (7)

QUESTION 7

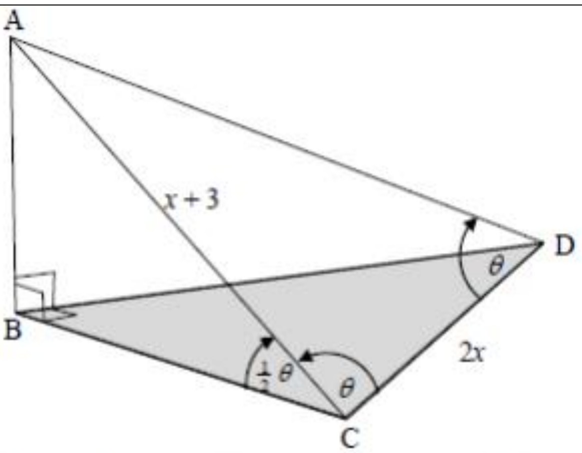
A corner of a rectangular block of wood is cut off and shown in the diagram below.
The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$.
Also $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.

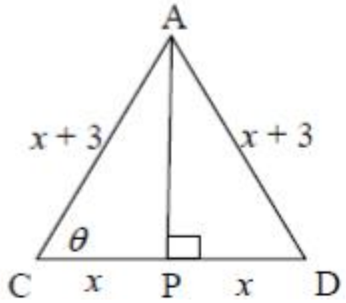


7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

7.2 Prove that $\cos\theta = \dfrac{x}{x+3}$. (4)

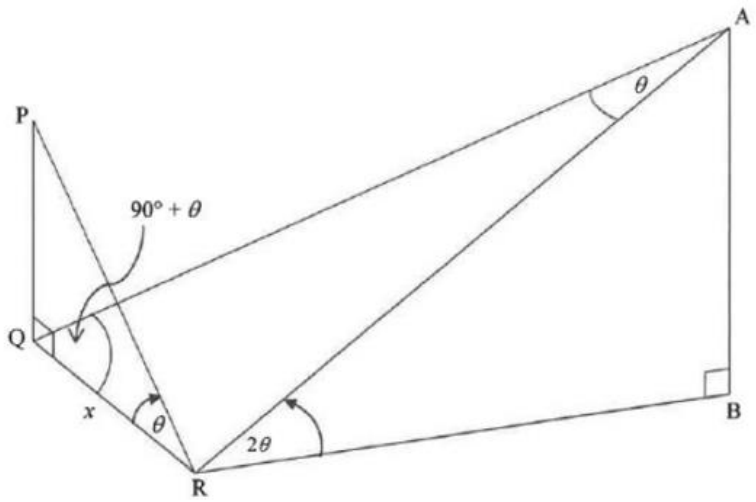
7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5)
[10]

QUESTION 7		
7.1	 $C\hat{A}D = 180° - 2\theta$ [∠s sum of Δ/∠e som van Δ]	

7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$ **OR/*OF*** AD $= x+3$ [sides opp = ∠s/*sye to* = ∠*e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$ **OR/*OF*** Draw/*Trek* AP ⊥ CD $\cos\theta = \frac{x}{x+3}$  	
7.3	$\cos\theta = \frac{2}{5}$ $\therefore \theta = 66{,}42°$ In ΔABC: $\sin\frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore$ AB $= 5\sin 33{,}21°$ $= 2{,}74$	

QUESTION 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



6.1 Determine in terms of x and θ:

6.1.1 QP (2)

6.1.2 AR (2)

6.2 Show that $AB = 2x\cos^2\theta$ (4)

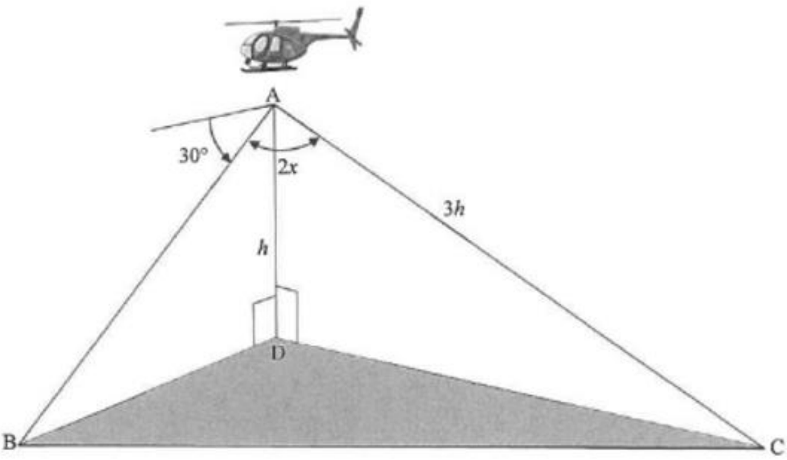
6.3 Determine $\frac{AB}{QP}$ if $\theta = 12°$. (2)

[10]

MARCH 2018	NOVEMBER 2018
Q6.1.1 $\tan\theta = \frac{PQ}{QR} = \frac{PQ}{x}$ $\therefore PQ = x\tan\theta$ **Q6.1.2** $\frac{AR}{\sin A\hat{Q}R} = \frac{QR}{\sin Q\hat{A}R}$ $AR = \frac{x\sin(90° + \theta)}{\sin\theta}$ **Q6.2** $\sin 2\theta = \frac{AB}{AR}$ $AB = AR\sin 2\theta$ $= \frac{x\sin(90° + \theta).\sin 2\theta}{\sin\theta}$ $= \frac{x\cos\theta.\sin 2\theta}{\sin\theta}$ $= \frac{x\cos\theta.2\sin\theta\cos\theta}{\sin\theta}$ $= 2x\cos^2\theta$ **Q6.3** $\frac{AB}{QP} = \frac{2x\cos^2 12°}{x\tan 12°}$ $= 9$	**Q7.1** $A\hat{B}D = 30°$ $\sin 30° = \frac{h}{AB}$ $AB = \frac{h}{\sin 30°}$ AB = 2h **Q7.2** $BC^2 = AB^2 + AC^2 - 2AB.AC\cos B\hat{A}C$ $= (2h)^2 + (3h)^2 - 2(2h)(3h)\cos 2x$ $= 13h^2 - 12h^2(2\cos^2 x - 1)$ $= 13h^2 - 24h^2\cos^2 x + 12h^2$ $= 25h^2 - 24h^2\cos^2 x$ $BC = h\sqrt{25 - 24\cos^2 x}$

QUESTION 7

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.



7.1 Determine the distance AB in terms of h. (2)

7.2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$. (4)

[6]